OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

4723

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

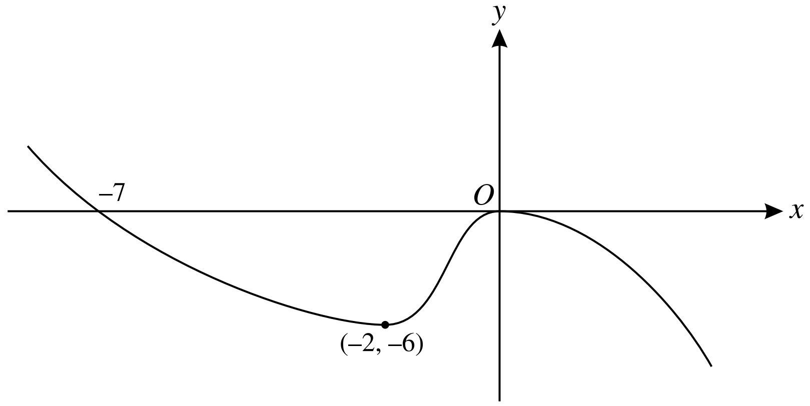
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Turn over

1 Solve the equation $|3x + 4a| = 5a$, where a is a positive constant. **[3]**

2



The diagram shows the curve with equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}(-7) = 0$ and that there are stationary points at $(-2, -6)$ and $(0, 0)$. Sketch the curve with equation $y = -4\mathrm{f}(x + 3)$, indicating the coordinates of the stationary points. **[4]**

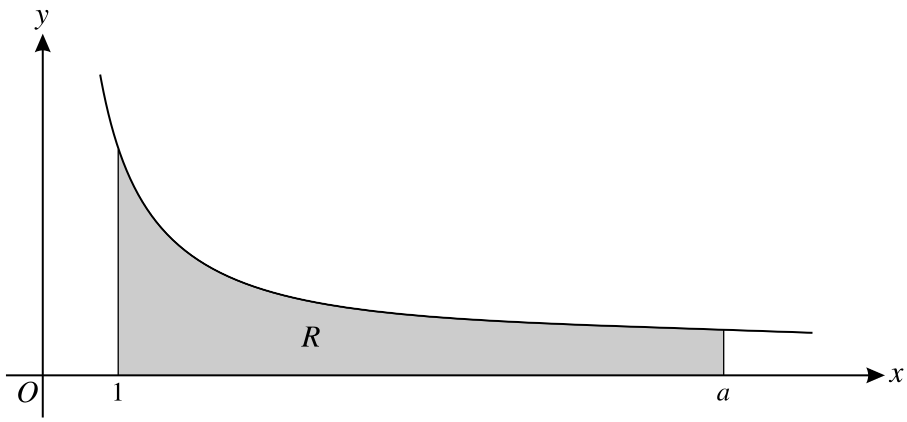
3 A giant spherical balloon is being inflated in a theme park. The radius of the balloon is increasing at a rate of 12 cm per hour. Find the rate at which the surface area of the balloon is increasing at the instant when the radius is 150 cm. Give your answer in cm^2 per hour correct to 2 significant figures.

[Surface area of sphere = $4\pi r^2$.] **[3]**

4 **(i)** Express $24\sin\theta + 7\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence solve the equation $24\sin\theta + 7\cos\theta = 12$ for $0° < \theta < 360°$. **[4]**

5



The diagram shows the curve with equation $y = \dfrac{6}{\sqrt{3x-2}}$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 1$, $x = a$ and $y = 0$, where a is a constant greater than 1. It is given that the area of R is 16 square units. Find the value of a and hence find the exact volume of the solid formed when R is rotated completely about the x-axis. **[9]**

6 The curve with equation $y = \dfrac{3x+4}{x^3 - 4x^2 + 2}$ has a stationary point at P. It is given that P is close to the point with coordinates (2.4, −1.6).

(i) Find an expression for $\dfrac{dy}{dx}$ and show that the x-coordinate of P satisfies the equation

$$x = \sqrt[3]{\tfrac{16}{3}x + 1}.$$ **[4]**

(ii) By first using an iterative process based on the equation in part **(i)**, find the coordinates of P, giving each coordinate correct to 3 decimal places. **[5]**

7 The function f is defined for $x > 0$ by $f(x) = \ln x$ and the function g is defined for all real values of x by $g(x) = x^2 + 8$.

(i) Find the exact, positive value of x which satisfies the equation $fg(x) = 8$. **[3]**

(ii) State which one of f and g has an inverse and define that inverse function. **[3]**

(iii) Find the exact value of the gradient of the curve $y = gf(x)$ at the point with x-coordinate e^3. **[3]**

(iv) Use Simpson's rule with four strips to find an approximate value of

$$\int_{-4}^{4} fg(x)\,dx,$$

giving your answer correct to 3 significant figures. **[3]**

8 **(a)** **(i)** Sketch the graph of $y = \operatorname{cosec} x$ for $0 < x < 4\pi$. **[3]**

(ii) It is given that $\operatorname{cosec}\alpha = \operatorname{cosec}\beta$, where $\frac{1}{2}\pi < \alpha < \pi$ and $2\pi < \beta < \frac{5}{2}\pi$. By using your sketch, or otherwise, express β in terms of α. **[2]**

(b) **(i)** Write down the identity giving $\tan 2\theta$ in terms of $\tan\theta$. **[1]**

(ii) Given that $\cot\phi = 4$, find the exact value of $\tan\phi \cot 2\phi \tan 4\phi$, showing all your working. **[6]**

[Question 9 is printed overleaf.]

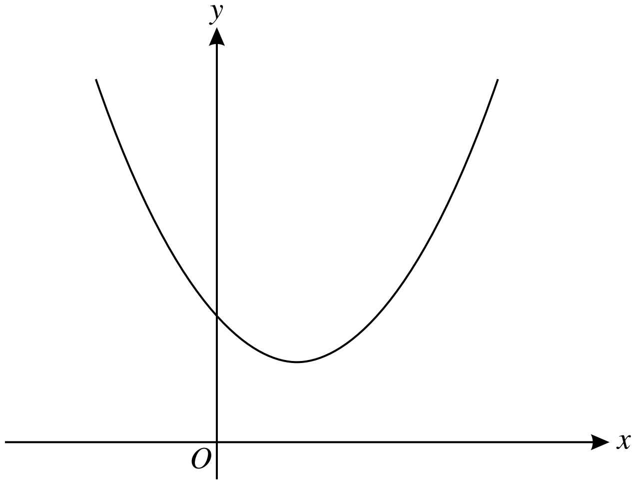
9 **(i)** The function f is defined for all real values of x by

$$f(x) = e^{2x} - 3e^{-2x}.$$

(a) Show that $f'(x) > 0$ for all x. **[3]**

(b) Show that the set of values of x for which $f''(x) > 0$ is the same as the set of values of x for which $f(x) > 0$, and state what this set of values is. **[5]**

(ii)



The function g is defined for all real values of x by

$$g(x) = e^{2x} + ke^{-2x},$$

where k is a constant greater than 1. The graph of $y = g(x)$ is shown above. Find the range of g, giving your answer in simplified form. **[5]**

1	Either: Obtain $\frac{1}{3}a$	B1		condone $\lvert x\rvert = \frac{1}{3}a$
	Attempt solution of linear eqn	M1		with signs of $3x$ and $5a$ different; allow M1 only if a given particular value and no recovery occurs; allow M1 only if a in terms of x attempted; allow M1 only if double inequality attempted but with no recovery to state actual values of x
	Obtain $-3a$	A1	**3**	as final answer
	Or: Obtain $9x^2 + 24ax + 16a^2 = 25a^2$	B1		
	Attempt solution of 3-term quad eqn	M1		as far as substitution into correct quadratic formula or correct factorisation of their quadratic; allow M1 only if a given particular value
	Obtain $-3a$ and $\frac{1}{3}a$	A1	**(3)**	or equivs; as final answers; and no others
			$\boxed{3}$	
2	Draw graph showing reflection in a horizontal axis	M1		
	Draw graph showing translation	M1		parallel to x-axis, in either direction; independent of first M1; not earned if curve still passes through O but ignore other coordinates given at this stage
	Draw (more or less) correct graph which must at least reach the negative x-axis, if not cross it, at left end of curve	A1		but ignoring no or wrong stretch in y-dir'n; condone graph existing only for $x < 0$; consider shape of curve and ignore coordinates given
	State $(-5, 24)$ and $(-3, 0)$ wherever located	B1	**4**	or clearly implied by sketch; allow for coordinates whatever sketch looks like; allow if in solution with no sketch
			$\boxed{4}$	
3	Either: State or imply $8\pi r$ as derivative	B1		or equiv
	Attempt to connect 12 and their derivative	M1		numerical or algebraic; using multiplication or division
	Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	**3**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
	Or: Use $r = 12t$ to show $S = 576\pi t^2$	B1		
	Attempt $\frac{dS}{dt}$ and substitute for t	M1		
	Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	**(3)**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
			$\boxed{3}$	

4	(i)	Obtain $R = 25$	B1		allow $\sqrt{625}$ or value rounding to 25
		Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin\alpha = 7$, $\cos\alpha = 24$ in the working
		Obtain $16.3°$	A1	3	or greater accuracy 16.260…; must be degrees now; allow $16°$ here
	(ii)	Show correct process for finding one answer	M1		even if leading to answer outside 0 to 360
		Obtain (28.69 – 16.26 and hence) $12.4°$	A1		or greater accuracy 12.425… or anything rounding to 12.4
		Show correct process for finding second answer	M1		even if further incorrect answers produced
		Obtain (151.31 – 16.26 and hence) $135°$ or $135.1°$	A1	4	or greater accuracy 135.054…; and no other between 0 and 360
		[SC: No working shown and 2 correct angles stated - B1 only in part (ii)]			
				$\boxed{7}$	

5		Integrate to obtain form $k(3x-2)^{\frac{1}{2}}$	M1		any non-zero constant k; or equiv involving substitution
		Obtain correct $4(3x-2)^{\frac{1}{2}}$	A1		or (unsimplified) equiv such as $\frac{6(3x-2)^{\frac{1}{2}}}{3\times\frac{1}{2}}$
		Apply limits and attempt solution for a	M1		assuming integral of form $k(3x-2)^n$; taking solution as far as removal of root; with subtraction the right way round; if sub'n used, limits must be appropriate
		Obtain $a = 9$	A1		(this answer written down with no working scores 0/4 so far but all subsequent marks are available)
		State or imply formula $\int \frac{36\pi}{3x-2}\,dx$	B1		or (unsimplified) equiv; condone absence of dx; allow B1 retroactively if π absent here but inserted later
		Integrate to obtain form $k\ln(3x-2)$	*M1		any constant k including π or not; condone absence of brackets
		Obtain $12\pi\ln(3x-2)$ or $12\ln(3x-2)$	A1√		following their integral of form $\int \frac{k}{3x-2}\,dx$
		Apply limits the correct way round	M1		dep *M; use of limit 1 is implied by absence of second term; allow use of limit a
		Obtain $12\pi\ln 25$ (or $24\pi\ln 5$)	A1	9	or exact equiv but not with $\ln 1$ remaining; condone answers such as $\pi 12\ln 25$ and $12\ln 25\pi$
				$\boxed{9}$	

6	**(i)**	Attempt use of quotient rule	M1		or equiv; allow numerator wrong way round but needs minus sign in numerator; for M1 condone 'minor' errors such as sign slips, absence of square in denominator, and absence of some brackets
		Obtain $\dfrac{3(x^3-4x^2+2)-(3x+4)(3x^2-8x)}{(x^3-4x^2+2)^2}$	A1		or equiv; allow A1 if brackets absent from $3x+4$ term or from $3x^2-8x$ term but not from both
		Equate numerator to 0 and attempt simplification	M1		at least as far as removing brackets, condoning sign or coeff slips; or equiv
		Obtain $-6x^3+32x+6=0$ or equiv and hence $x=\sqrt[3]{\frac{16}{3}x+1}$	A1	**4**	AG; necessary detail needed (i.e. at least one intermediate step) and following first derivative with correct numerator
	(ii)	Obtain correct first iterate having used initial value 2.4	B1		showing at least 3 dp (2.398 or 2.399 or greater accuracy 2.39861…)
		Apply iterative process	M1		to obtain at least 3 iterates in all; implied by plausible, converging sequence of values; having started with any initial non-negative value
		Obtain at least 3 correct iterates from their starting point	A1		allowing recovery after error
		Obtain 2.398	A1		value required to exactly 3 dp
		Obtain –1.552	A1	**5**	value required to exactly 3 dp; allow if apparently obtained by substitution of 2.4; answers only with no iterates shown gets 0/5

$[\,2.4 \;\rightarrow\; 2.3986103 \;\rightarrow\; 2.3981808 \;\rightarrow\; 2.3980480\,]$

$\boxed{9}$

7	**(i)**	State $\ln(x^2+8)=8$	B1		or equiv such as $x^2+8=e^8$
		Attempt solution involving e^8	M1		by valid (exact) method at least as far as $x^2=...$
		Obtain $\sqrt{e^8-8}$	A1	**3**	or exact equiv; and no other answer
	(ii)	State f only	B1		
		State e^x or e^y	B1		or equiv; allow if g, or f and g, chosen
		Indicate domain is all real numbers	B1	**3**	however expressed
	(iii)	Attempt use of chain rule	M1		whether applied to gf or fg; or equiv such as use of product rule on $(\ln x)(\ln x)+8$
		Obtain $\frac{2\ln x}{x}$	A1		or equiv
		Obtain $6e^{-3}$	A1	**3**	or exact equiv but not including ln
	(iv)	Attempt evaluation using *y* attempts	M1		with coeffs 1, 4 and 2 occurring at least once each; whether fg or gf
		Obn $k(\ln 24+4\ln 12+2\ln 8+4\ln 12+\ln 24)$	A1		any constant *k*
		Use $k=\frac{2}{3}$ and obtain 20.3	A1	**3**	or greater accuracy (20.26…) but must round to 20.3

[Note that use of Simpson's rule between 0 and 4 with two strips, coeffs 1, 4, 1, followed by doubling of result is equiv;
SC: Use of Simpson's rule between 0 and 4 with four strips followed by doubling of result - allow 3/3 - answer is 20.2 (20.2327…)]

$\boxed{12}$

8 (a) (i) Draw at least two correctly shaped branches, one for $y > 0$, one for $y < 0$	M1		otherwise located anywhere including $x < 0$
Draw four correct branches	M1		now (more or less) correctly located;
Draw (more or less) correct graph	A1	**3**	with some indication of horiz scale (perhaps only 4π indicated); with asymptotic behaviour shown (but not too fussy about branch drifting slightly away from asymptotic value nor about branch touching asymptote) but branches must not obviously cross asymptotic value; with –1 and 1 shown (or implied by presence of sine curve or by presence of only one of them on a reasonably accurate sketch); no need for vertical (dotted) lines drawn to indicate asymptotic values
(ii) State expression of form $k\pi + \alpha$ or $k\pi - \alpha$ or $\alpha = k\pi + \beta$ or $\alpha = k\pi - \beta$	M1		any non-zero numerical value of *k*; M0 if degrees used
State $3\pi - \alpha$	A1	**2**	or unsimplified equiv
(b) (i) State $\dfrac{2\tan\theta}{1-\tan^2\theta}$	B1	**1**	or equiv such as $\dfrac{t+t}{1-t\times t}$ or $\dfrac{2\tan A}{1-\tan^2 A}$
(ii) State or imply $\tan\phi = \frac{1}{4}$	B1		or equiv such as $\frac{1}{\tan\phi} = 4$
Attempt to evaluate $\tan 2\phi$ or $\cot 2\phi$	M1		perhaps within attempt at complete expression but using correct identity
Obtain $\tan 2\phi = \frac{8}{15}$ or $\cot 2\phi = \frac{15}{8}$	A1		or (unsimplified) equiv; may be implied
Attempt to evaluate value of $\tan 4\phi$	M1		perhaps within attempt at complete expression; condone only minor slip(s) in use of relevant identity
Obtain $\frac{240}{161}$	A1		or (unsimplified) exact equiv; may be implied
Obtain final answer $\frac{225}{322}$	A1	**6**	or exact equiv

[SC – (use of calculator and little or no working)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\tan 2\phi = \frac{8}{15}$ B1; Obtain $\frac{225}{322}$ B1 (max 3/6)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\frac{225}{322}$ B2 (max 3/6)

$\boxed{12}$

9 (i)	(a) Differentiate to obtain $k_1e^{2x} + k_2e^{-2x}$	M1		any constants k_1 and k_2 but derivative must be different from f(x); condone presence of + c
	Obtain $2e^{2x} + 6e^{-2x}$	A1		or unsimplified equiv; no + c now
	Refer to $e^{2x} > 0$ and $e^{-2x} > 0$ or to more general comment about exponential functions	A1	3	or equiv (which might be sketch of $y = f(x)$ with comment that gradient is positive or might be sketch of $y = f'(x)$ with comment that $y > 0$; AG
	(b) Differentiate to obtain $k_3e^{2x} + k_4e^{-2x}$	M1		any constants k_3 and k_4 but second derivative must be different from their first derivative; condone presence of + c
	Obtain $4e^{2x} - 12e^{-2x}$	A1		or unsimplified equiv; no + c now
	Attempt solution of $f''(x) > 0$ or of $f(x) > 0$ or of corresponding eqn	M1		at least as far as term involving e^{4x} or e^{-4x}
	Obtain $x > \frac{1}{4}\ln 3$	A1		
	Confirm both give same result	B1	5	AG; necessary detail needed; either by solving the other or by observing that same inequality involved (just noting that $f''(x) = 4f(x)$ is sufficient)
(ii)	Differentiate to obtain $2e^{2x} - 2ke^{-2x}$	B1		or unsimplified equiv
	Attempt to find x-coordinate of stationary pt	M1		equating to 0 and reaching $e^{4x} = ...$ or equiv
	Obtain $e^{4x} = k$ and hence $\frac{1}{4}\ln k$ or equiv	A1		or equiv such as $e^{2x} = \sqrt{k}$
	Substitute and attempt simplification	M1		using valid processes but allow if only limited progress [note that question can be successfully concluded (without actually finding x) by substitution of $e^{2x} = \sqrt{k}$ and $e^{-2x} = \frac{1}{\sqrt{k}}$]
	Obtain $g(x) \geq 2\sqrt{k}$ or $y \geq 2\sqrt{k}$	A1	5	or similarly simplified equiv with $\geq$ not >
			$\boxed{13}$	